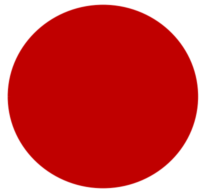
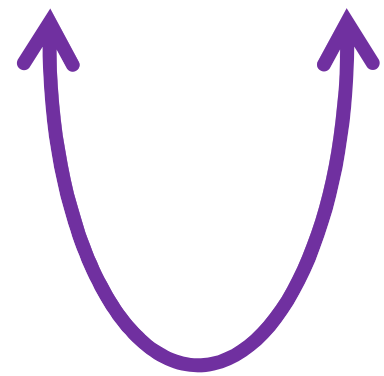
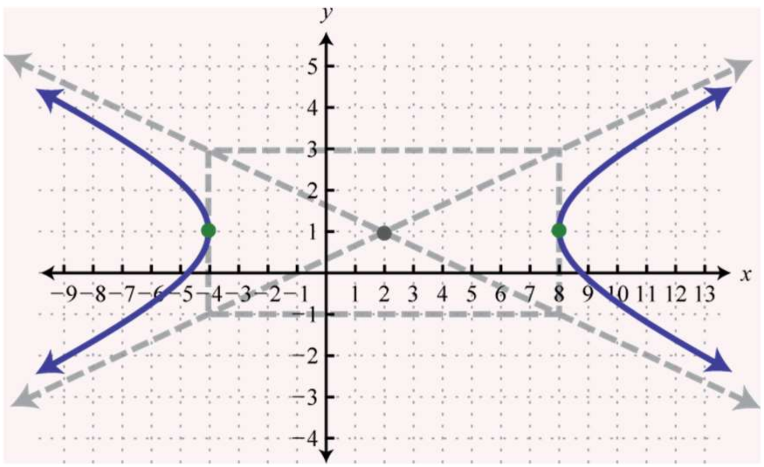
Ch. 11 Shifted Conics





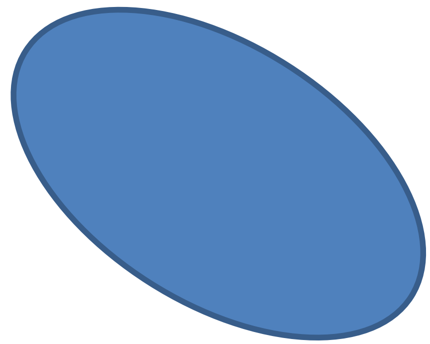
Circle

Parabola



Ellipse

Hyperbola



Using the sheet provided in class, fill in the blanks and solve #1-10 at bottom of page.

REMINDERS FOR CONICS:

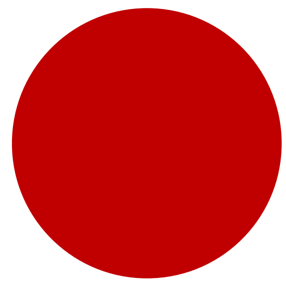
Circle: TWO squared terms with EQUAL coefficients, both are POSITIVE

$$(x-h)^2+(y-k)^2=r^2$$

key values for graphing:

(h, k) = center

r = radius



Ellipse: TWO squared terms with DIFFERENT coefficients, both are POSITIVE

$$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1 \qquad \frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$$

a^2 is always the LARGEST denominator for ellipse and will dictate the horizontal/vertical orientation

key values for graphing:

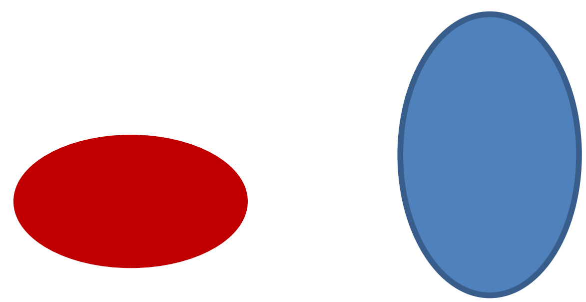
(h, k) = center

a is the distance from center to each vertex

2a = MAJOR axis (contains **both** major **vertices**)

2b = MINOR axis

foci: $c^2 = a^2 - b^2$



Hyperbola: TWO squared terms, one term is NEGATIVE due to subtraction

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1 \qquad \frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

a^2 is always with the POSITIVE term for a hyperbola and will dictate the horizontal/vertical orientation.

key values for graphing:

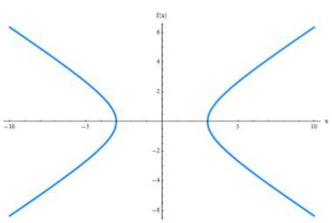
(h, k) = center

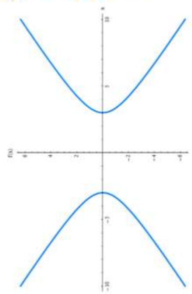
a is the distance from center to each vertex

2a = transverse axis (contains **both vertices**)

use a and b to sketch central box & asymptotes

foci: $c^2 = a^2 + b^2$





Parabola: ONE squared term (one vertex)

$$(x-h)^2 = 4p(y-k) \qquad\qquad (y-k)^2 = 4p(x-h)$$

key values for graphing:

(h, k) = center

p = distance from vertex to **focus** point and from vertex to **directrix** line

4p = **focal diameter**

Today's assignment:

Mixed Conics #1-10

identify conic,
write in standard form,
sketch graph.

Show work!

(foci not necessary)

Only find and label the key values/points needed for sketching each graph.

TODAY'S ASSIGNMENT: Mixed Conics

IDENTIFY the conic, complete the square to write each equation in STANDARD FORM, then GRAPH it. Only identify key values needed for graphing such as the center, vertex, radius, and the a, b, or p values.

1. $x^2 - 4y - 6x + 9 = 0$ Parabola
2. $x^2 - 8x + y^2 + 6y + 24 = 0$ Circle
3. $x^2 - 3y^2 + 2x - 24y - 41 = 0$ Hyperbola
4. $9x^2 + 25y^2 - 54x - 50y - 119 = 0$ Ellipse
5. $x^2 = y + 8x - 16$
6. $x^2 - 4x - y^2 - 5 - 4y = 0$
7. $5x^2 + 2y^2 - 40x - 20y + 110 = 0$
8. $x^2 - 8x + 11 = -y^2$
9. $8y^2 - 9x^2 - 16y + 36x - 100 = 0$
10. $4y^2 + 4y + 8x = 15$

CHECK ANSWERS:

parabola parabola parabola ellipse ellipse
hyperbola hyperbola hyperbola circle circle

$$\left(y+\frac{1}{2}\right)^2=-2(x-2)$$

$$\frac{(y-1)^2}{9}-\frac{(x-2)^2}{8}=1$$

$$(x-4)^2+(y+3)^2=1$$

$$\frac{(y+4)^2}{2}-\frac{(x+1)^2}{6}=1$$

$$(x-4)^2+y^2=5$$

$$\frac{(x-2)^2}{5}-\frac{(y+2)^2}{5}=1$$

$$(x-4)^2=y$$

$$\frac{(x-3)^2}{25}+\frac{(y-1)^2}{9}=1$$

$$(x-3)^2=4y$$

$$\frac{(y-5)^2}{10}+\frac{(x-4)^2}{4}=1$$

Mixed Conics

(a) Identify the **conic.** Parabola

(b) Complete square, write equation in standard form.

(c) Sketch graph using **key values.**

1. $x^2 - 4y - 6x + 9 = 0$

$(x^2 - 6x + 9) = 4y - 9 + 9$

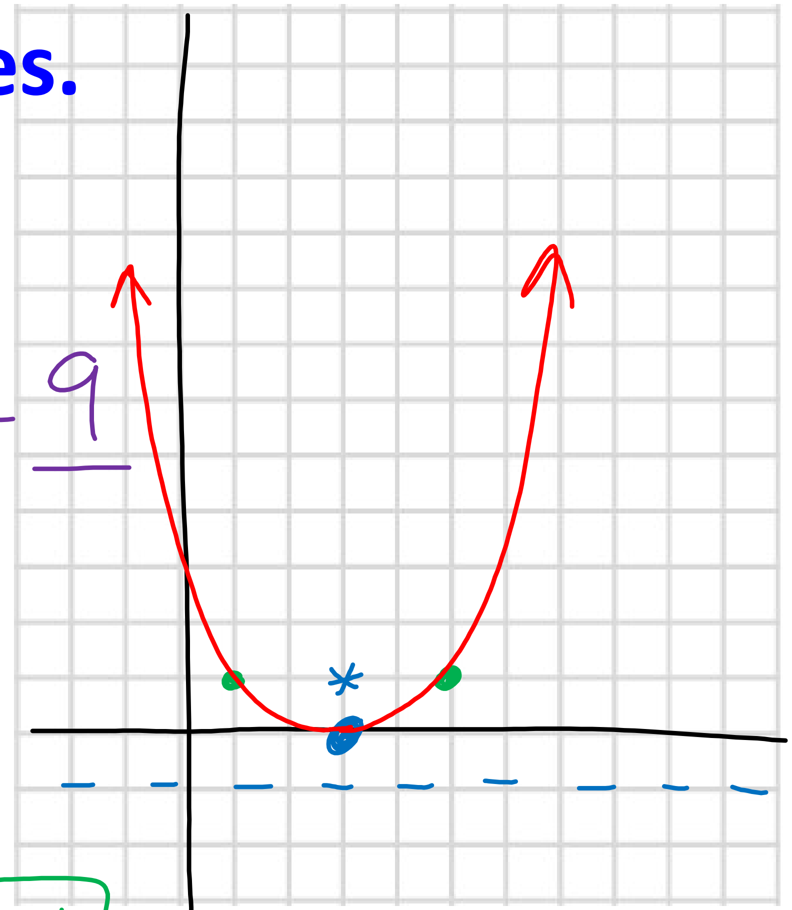
$\left(\frac{-6}{2}\right)^2$

$\boxed{(x - 3)^2 = 4(y + 0)}$

Vertex $(h, k) = (3, 0)$

$4p = 4$ $\boxed{\text{focal diameter} = 4}$

$p = 1$ $\boxed{\text{up 1 to focus, down 1 to directrix}}$



$$2.\ \boxed{x^2} - 8x + \boxed{y^2} + 6y + 24 = 0$$

Circle ✓

$$x^2 - 8x + \underline{16} + y^2 + 6y + \underline{9} = -24 + \underline{16} + \underline{9}$$

$$(x-4)^2 + (y+3)^2 = \underset{r^2}{1}$$

radius = r

center $(h,k) = (4,-3)$

3. $\boxed{x^2 - 3y^2} + 2x - 24y - 41 = 0$ hyperbola ✓

$= 1$

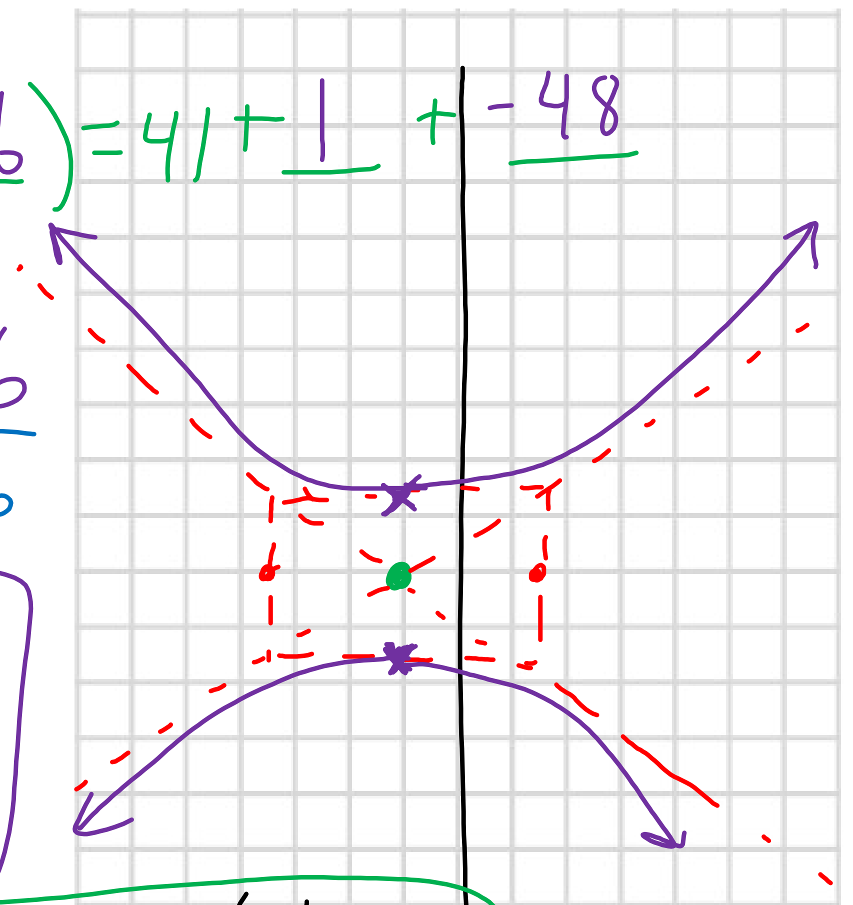
$$x^2 + 2x - 3y^2 - 24y = 41$$

$$\underbrace{x^2 + 2x + \underline{1}} - 3\underbrace{(y^2 + 8y + \underline{16})} = 41 + \underline{1} + \underline{-48}$$

$$\frac{(x+1)^2}{-6} + \frac{\overset{1}{\cancel{3}}(y+4)^2}{+\underset{2}{\cancel{6}}} = \frac{-6}{-6}$$

$$\boxed{\frac{(y+\overset{k}{4})^2}{2} - \frac{(x+\overset{h}{1})^2}{6} = 1}$$

a^2 b^2



$\boxed{\text{center} = (-1, -4)}$
h, k

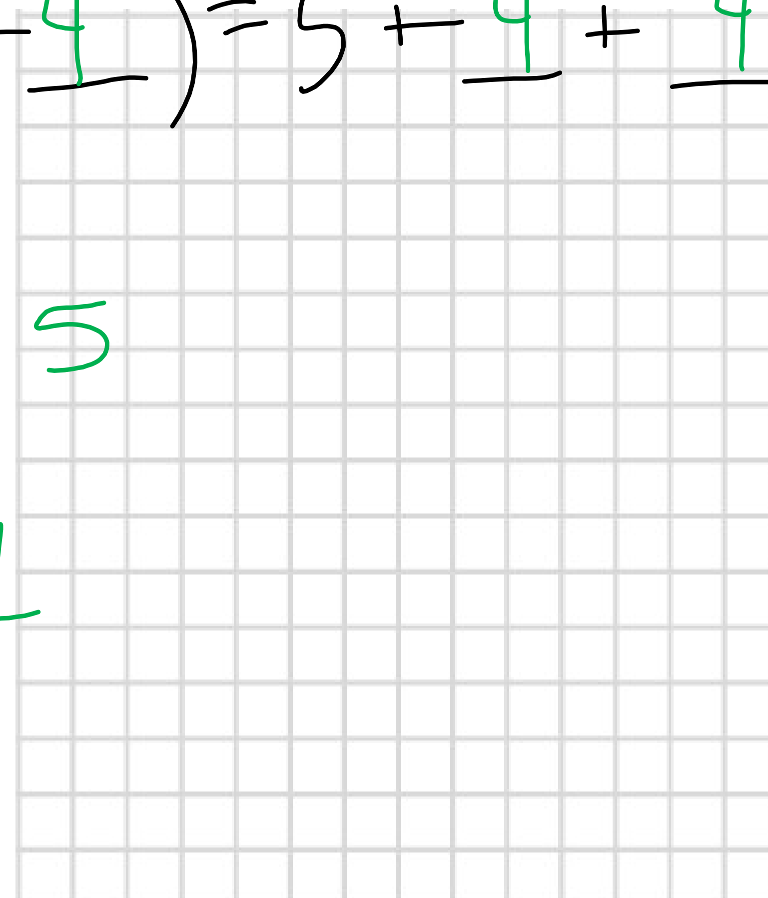
6. $x^2 - 4x - y^2 - 5 - 4y = 0$

$$x^2 - 4x + \underline{\quad} - 1y^2 - 4y = 5$$

$$x^2 - 4x + \underline{4} - 1\left(y^2 + 4y + \underline{4}\right) = 5 + \underline{4} + \underline{-4}$$

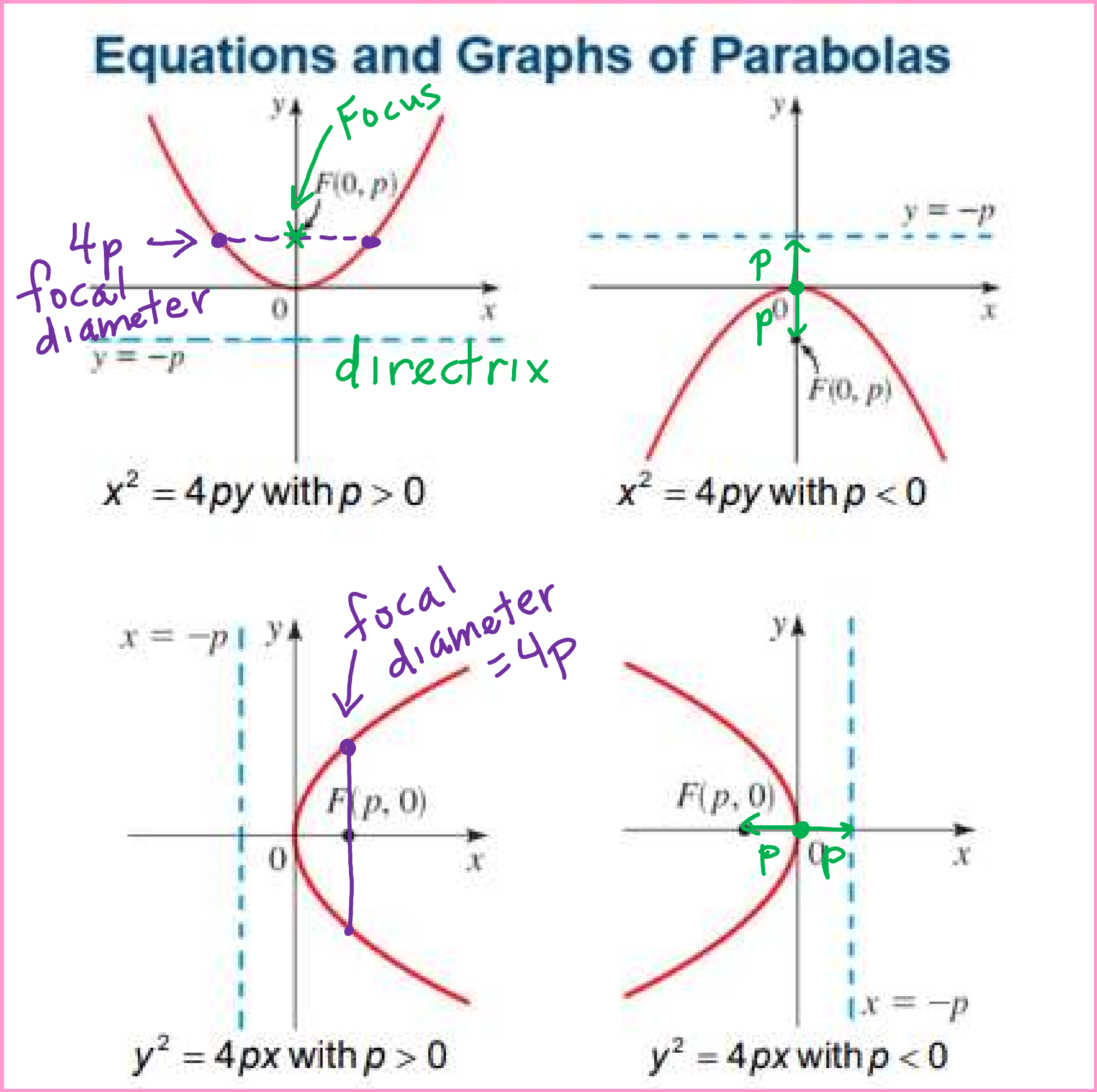
$$(x-2)^2 - (y+2)^2 = 5$$

$$\frac{(x-2)^2}{5} - \frac{(y+2)^2}{5} = 1$$



Refer to pink sheet if you need help identifying key points and/or sketching the graph.

11.1 notes previously added to pink sheet:



Eccentricity: $e = \dfrac{c}{a}$ **Ellipses**

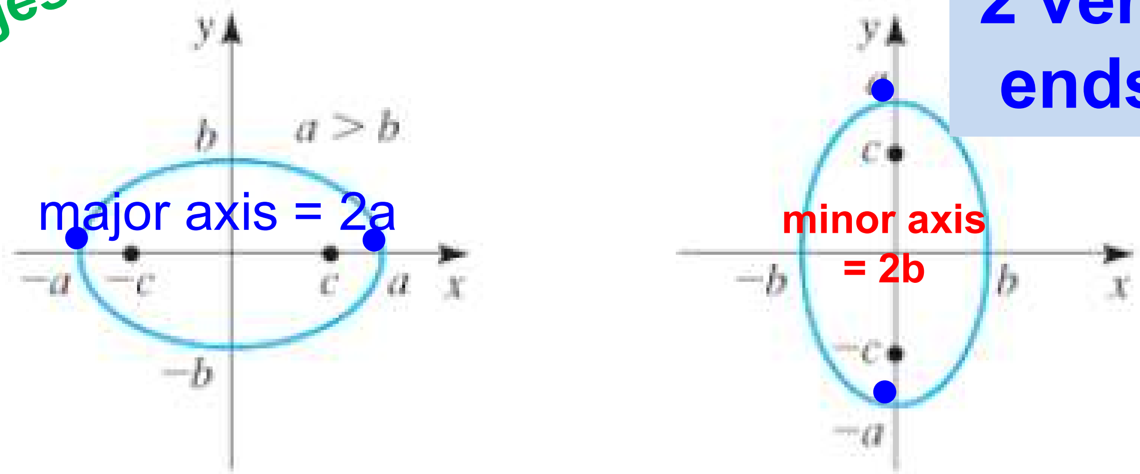
horizontal $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

largest→

vertical

$\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2}$ ← largest

2 vertices always at ends of major axis



Foci $(\pm c, 0)$, $c^2 = a^2 - b^2$

Foci $(0, \pm c)$, $c^2 = a^2 - b^2$

$$c^2 = a^2 - b^2$$

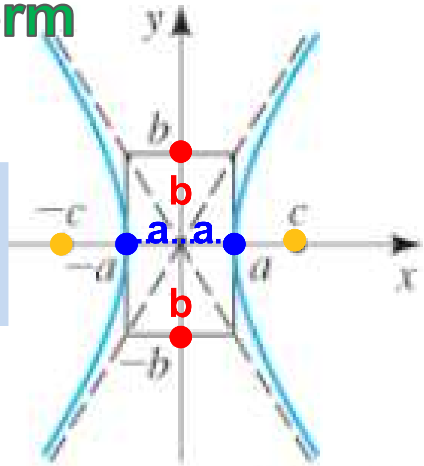
2 foci located on major axis "c" units from the center

11.2 notes previously added to pink sheet

$y = \pm\frac{b}{a}x$ **Asymptotes** $y = \pm\frac{a}{b}x$

Horizontal orientation because x term is positive

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

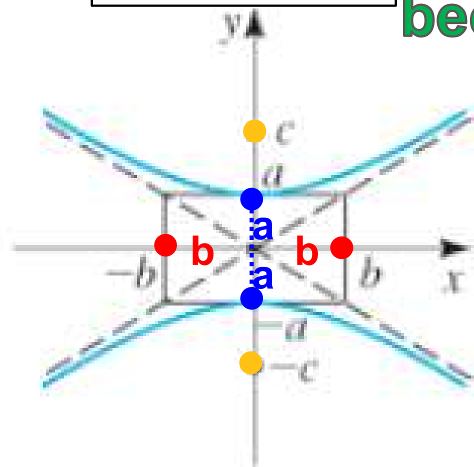


Foci $(\pm c, 0)$, $c^2 = a^2 + b^2$

transverse axis = 2a

Vertical orientation because y term is positive

$$\boxed{\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1}$$



Foci $(0, \pm c)$, $c^2 = a^2 + b^2$

2 vertices always at ends of the transverse axis

$$\boxed{c^2 = a^2 + b^2}$$

2 foci located on transverse axis "c" units from the center

11.3 notes previously added to pink sheet